GRB neutrinos, Lorenz Invariance Violation and the influence of background cosmology

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Abstract

Modern ideas in quantum gravity predict the possibility of Lorenz Invariance Violation (LIV) manifested e.g. by energy dependent modification of standard relativistic dispersion relation. In a recent paper Jacob and Piran proposed that time of flight delays in high energy neutrinos emitted by gamma ray bursts (GRBs) located at cosmological distances can become a valuable tool for setting limits on LIV theories. However, current advances in observational cosmology suggest that our Universe is dominated by dark energy with relatively little guidance on its nature thus leading to several cosmological scenarios compatible with observations.

In this paper we raise the issue of how important, in the context of testing LIV theories, is our knowledge of background cosmological model. Specifically we calculate expected time lags for high-energy (100 TeV) neutrinos in different cosmological models. Out of many particular models of dark energy we focus on five: ΛCDM, quintessence, quintessence with time varying equation of state, brane-world and generalized Chaplygin gas model as representative for various competing approaches.

The result is that uncertainty introduced by our ignorance concerning the right phenomenological model describing dark energy dominated universe is considerable and may obscure bounds derived from studying time delays from cosmological sources.

Keywords: gamma ray bursts, high energy neutrinos, dark energy theory

1 Introduction

Modern approaches to quantum gravity predict the Lorenz Invariance Violation (LIV thereafter) manifesting itself in particular as an energy dependent modification of relativistic dispersion relation. Essentially, additional terms in dispersion relation follow a power law expansion with respect to E/E_{QG} where E denotes the particle's (photon's) energy and E_{QG} is the quantum gravity energy scale. The first guess concerning E_{QG} would be to assume it being of order of the Planck energy $E_{Pl} = 1.2\ 10^{19}\ GeV$. Although there are suggestions that in some concrete models (e.g. with large extra dimensions) E_{QG} could be considerably lower than Planck energy, it is clear that departures from standard relativistic dispersion relation can be seen only in high-energy particles or photons.

Several years ago it has been proposed to use astrophysical objects to look for energy dependent time of arrival delays [1]. Specifically gamma ray bursts (GRBs) being energetic events visible from cosmological distances are the most promising sources of constraining LIV theories. Indeed they were already discussed in this context (quite recently in [2]) and even used to obtain some constraints [3]. One is facing here a problem that in the energy range typical for gamma ray photons the LIV effects are very subtle. On the other hand, one could imagine looking for TeV photons which would be produced by GRBs in synchrotron self Compton mechanism [4, 5]. However, the Universe filled with 2.7 K cosmic microwave background radiation becomes opaque, via pair production process, to photons with energies above 10 TeV. This is analogous to GZK threshold for particles. Despite the fact that LIV theories are often invoked to resolve the GZK paradox [6] and that 20 TeV photons were reported come from Mk 501 BL Lac object [7] the use of very high energy photons from GRBs can be tricky.

Hopefully in a recent paper Jacob and Piran proposed to use high energy neutrinos instead of photons [8]. Emission of $100 - 10^4\ TeV$ neutrinos is typically predicted in current models of GRBs [4] and as noticed in [8] the forthcoming neutrino detectors like Ice Cube are extremely quiet in this energy range. Measurements of time delay between prompt gamma ray photons and neutrino signal would open a new window on exploring LIV theories. Therefore this idea is worth further consideration. In this paper we discuss the sensitivity of this setting to the details of cosmological model.

The discovery of accelerated expansion of the Universe [9] introduced the problem of "dark energy" in the Universe which is now one of the most important issues in modern cosmology. A lot of specific scenarios have been put forward as an explanation of this puzzling phenomenon. They fall into two broad categories: searching an explanation among hypothetical candidates for dark energy (cosmological constant Λ [9], quintessence - evolving scalar fields [10], Chaplygin gas [11]) or modification of gravity theory (e.g. brane world scenarios [12]). We will examine the LIV induced time delays between prompt photon and neutrino arrivals in the above mentioned five classes of cosmological models.

In the next Section we briefly recall phenomenology of distorted dispersion relation in LIV theories and its consequences for energy dependent time of arrival delays. Section 2 also briefly outlines cosmological models considered. Note that brane-world scenario is not just a candidate for background cosmology, but also "represents" the class of theories in which LIV occurs [13]. The final section contains results and conclusions.

2 LIV induced time delays in different cosmological models

Following [8] and for better comparison of results we assume the modified dispersion relation for neutrinos from GRB sources in the form:

$$E_\nu^2 - p_\nu^2 c^2 - m_\nu^2 c^4 = \epsilon E_\nu^2 \left(\frac{E_\nu}{\xi_n E_{QG}} \right)^n \tag{1}$$

where:$\epsilon = \pm 1$ with $+1$ corresponding to superluminal and -1 to infraluminal motion, ξ_n is a dimensionless parameter. In order to get the results comparable with [8] we assume E_{QG} equal to the Planck energy, $\xi_1 = 1$ and $\xi_2 = 10^{-7}$. The dispersion relation (1) essentially corresponds to the power-law expansion (see [3]) so for the practical purposes (due to smallness of expansion parameter E/E_{QG}) only the lowest terms of the expansion are relevant. Because in some LIV theories the odd power terms might be forbidden [14] we retain the cases of $n = 1$ and $n = 2$.

The relation (1) leads to a hamiltonian of the following form

$$H = \sqrt{(p_\nu^2 c^2 + m_\nu^2 c^4)\,[1 + \epsilon \left(\frac{E_\nu}{\xi_n E_{QG}} \right)^n]} \tag{2}$$

Because of the expansion of the Universe, neutrino momentum $p_\nu = p_\nu(t)$ is related to the cosmic scale factor $a(t)$ through

$$p_\nu(t) = \frac{p_\nu(t_0)}{a(t)} \tag{3}$$

Later on the scale factor will be re-expressed in terms of redshift z which is observable quantity. Similar relation holds of course for neutrino energy $E_\nu = E_\nu(t)$.
The time dependent velocity is given by

$$v(t) = \frac{\partial H}{\partial p} \tag{4}$$

From (4), using hamiltonian (2), dispersion relation (1) and scale factor dependence (3) one can easily obtain (to the lowest order in terms of the observed neutrino energy $E_\nu(t_0) \equiv E_{\nu 0}$) that

$$v_\nu(t) \simeq \frac{c}{a(t)}[1 - \frac{1}{2}\frac{m_\nu^2 c^4}{E_{\nu 0}^2} a^2(t) + \frac{1}{2}(n+1)\epsilon \left(\frac{E_{\nu 0}}{\xi_n E_{QG}} \right)^n \frac{1}{a^n(t)}] \tag{5}$$

The comoving distance travelled by neutrino from a GRB to the Earth is defined as

$$r(t) = \int_{t_{emission}}^{t_0} v(t) dt \tag{6}$$

Taking into account that $a(t) = \frac{1}{1+z}$ we can express the above relation (6) in terms of redshift z

$$r(z) = \int_0^z v(z) \frac{dz}{H(z)(1+z)} \tag{7}$$

where for neutrinos we have

$$v(z) \simeq c(1+z)[1 - \frac{1}{2}\frac{m_\nu^2 c^4}{E_{\nu 0}^2}\frac{1}{(1+z)^2} + \frac{1}{2}(n+1)\epsilon\left(\frac{E_{\nu 0}}{\xi E_{Pl}}\right)^n (1+z)^n] \tag{8}$$

and $H(z)$, as usually denotes the expansion rate. Time of flight for neutrinos (i.e. the comoving distance measured in light years) from GRB source to the Earth is then

$$t_\nu = \int_0^z [1 - \frac{m_\nu^2 c^4}{2E_{\nu 0}}\frac{1}{(1+z)^2} + \epsilon\frac{n+1}{2}\left(\frac{E_{\nu 0}}{\xi_n E_{QG}}\right)^n (1+z)^n]\frac{dz}{H(z)} \tag{9}$$

In the first term one easily recognizes the well known time of flight for prompt (lower energetic) photons so the time delay due to both, neutrino masses and LIV effects, between a high energy neutrino and a low energy prompt photon is equal to

$$\Delta t == \int_0^z [\frac{m_\nu^2 c^4}{2E_{\nu 0}}\frac{1}{(1+z)^2} - \epsilon\frac{n+1}{2}\left(\frac{E_{\nu 0}}{\xi_n E_{QG}}\right)^n (1+z)^n]\frac{dz}{H(z)} \tag{10}$$

In the calculations above we retained the neutrino mass — it is massive after all. For the purpose of further calculations we assume $m_\nu = 1\ eV$. However it is evident already from the formula (10) that the effect of non-zero mass of the neutrino is for our purpose negligible — in perfect accordance with formulas in [8].

The paragraphs below briefly introduce five types of cosmological models in which LIV induced time delays will be calculated. We will restrict our attention to flat models $k = 0$ because the flat FRW geometry is strongly supported by cosmic microwave background radiation (CMBR) data [16].

Friedman - Robertson - Walker model with non-vanishing cosmological constant and pressure-less matter including the dark part of it responsible for flat rotation curves of galaxies (the co called ΛCDM model) is a standard reference point in modern cosmology. Sometimes it is referred to as a concordance model since it fits rather well to independent data (such like CMBR data, LSS considerations, supernovae data). The cosmological constant suffers from the fine tuning problem (being constant, why does it start dominating at the present epoch?) and from the enormous discrepancy between facts and expectations (assuming that Λ represents quantum-mechanical energy of the vacuum it should be 55 orders of magnitude larger than observed [17]).

Hence another popular explanation of the accelerating Universe is to assume the existence of a negative pressure component called dark energy. One can heuristically assume that this component is described by hydrodynamical energy-momentum tensor with (effective) cosmic equation of state: $p = w\rho$ where $-1 < w < -1/3$ [18]. In such case this component is called "quintessence". Confrontation with supernovae and CMBR data [19] led to the constraint $w \leq -0.8$. This was further improved by combined analysis of SNIa and large scale structure considerations (see e.g. [20]) and from WMAP data on CMBR [21]. The most recent one comes from the ongoing ESSENCE supernova survey [22] and pins down the equation of state parameter w to the range $-1.07 \pm 0.09 (1\sigma) \pm 0.12 (systematics)$. For the illustrative purposes we chose $w = -0.87$ as representing a quintessence model which is different from cosmological constant and still admissible by the data.

If we think that the quintessence has its origins in the evolving scalar field, it would be natural to expect that w coefficient should vary in time, i.e. $w = w(z)$. An arbitrary

function $w(z)$ can be Taylor expanded. Then, bearing in mind that both SNIa surveys or strong gravitational lensing systems are able to probe the range of small and moderate redshifts it is sufficient to explore first the linear order of this expansion. Such possibility, i.e. $w(z) = w_0 + w_1 z$ has been considered in the literature (e.g. [23]). Fits to supernovae data performed in the literature suggest $w_0 = -1.5$ and $w_1 = 2.1$ [24] (which is consistent with fits given in [25]). Therefore we adapted these values as representative for this parametrization of the equation of state.

In the class of generalized Chaplygin gas models matter content of the Universe consists of pressure-less gas with energy density ρ_m representing baryonic plus cold dark matter (CDM) and of the generalized Chaplygin gas with the equation of state $p_{Ch} = -\frac{A}{{\rho_{Ch}}^{\alpha}}$ with $0 \leq \alpha \leq 1$, representing dark energy responsible for acceleration of the Universe. Using the angular size statistics for extragalactic sources combined with SNIa data it was found in [26] that in the the $\Omega_m = 0.3$ and $\Omega_{Ch} = 0.7$ scenario best fitted values of model parameters are $A_0 = 0.83$ and $\alpha = 1.$ respectively. Generalized Chaplygin gas models have been intensively studied in the literature [27] and in particular they have been tested against supernovae data (e.g. [28] and references therein). Conclusions from these fits are in agreement with the above mentioned values of parameters so we used them as representative of Chaplygin Gas models.

Brane-world scenarios assume that our four-dimensional spacetime is embedded into 5-dimensional space and gravity in 5-dimensions is governed by the usual 5-dimensional Einstein-Hilbert action. The bulk metric induces a 4-dimensional metric on the brane. The brane induced gravity models [12] have a 4-dimensional Einstein-Hilbert action on the brane calculated with induced metric. According to this picture, our 4-dimensional Universe is a surface (a brane) embedded into a higher dimensional bulk space-time in which gravity propagates. As a consequence there exists a certain cross-over scale r_c above which an observer will detect higher dimensional effects. Cosmological models in brane-world scenarios have been widely discussed in the literature [29]. It has been shown in [29] that flat brane-world Universe with $\Omega_m = 0.3$ and $r_c = 1.4\ H_0^{-1}$ is consistent with current SNIa and CMBR data. Note that in flat (i.e. $k = 0.$) brane-world Universe the following relation is valid: $\Omega_{r_c} = \frac{1}{4}(1 - \Omega_m)^2$. Futher research performed in [30] based on SNLS combined with SDSS disfavored flat brane-world models. More recent analysis by the same authors [31] using also ESSENCE supernovae sample and CMB acoustic peaks lead to the conclusion that flat brane-world scenario is only slightly disfavored, although inclusion of baryon acoustic oscillation peak would ruled it out. Despite this interesting debate we use flat brane-world scenario with $\Omega_m = 0.3$ for illustration.

Expansion rates $H(z) = \dot{a}/a$ (equivalent to Friedman equation) for the models studied are shown in Table 1.

3 Results and conclusions

We have calculated time delays of 100 TeV neutrinos as a function of redshift (see equation (10)) in different dark energy scenarios described above and for LIV theories with $n = 1$, $\xi_1 = 1$ and $n = 2$, $\xi_2 = 10^{-7}$ respectively. They are summarized in Figure 1. Redshift range from $z = 0$ to $z = 6$ represents the depth of GRB surveys [4] and hence reflects the range of distances from which one might expect the high energy neutrinos to come.

For better resolution we displayed the same information in Figure 2 but in a restricted range of redshifts. GRB sample with measured redshifts has a mode at about $z \sim 1.5$. Therefore a range from $z = 2$ to $z = 3$ in some sense also represents the most likely distance from a potential source of high energy neutrinos. Figure 3, at last, displays the energy dependence of time of flight delay for the source located at $z = 3$.

One can see noticeable differences between time delays calculated for different background cosmologies. ΛCDM model and quintessence model (with w parameter best fitted to current SNIa and CMBR data) introduce negligible confusion to time delays. Brane world models (i.e. the class representative of theories in which LIV is expected) and Chaplygin gas scenario predict time delays considerably lower than in ΛCDM cosmology. For example the differences in time delays of 100 TeV neutrino from a source at $z = 3$ between ΛCDM and Chaplygin gas model is almost 3 hours for $n = 2$ LIV theories and 43 minutes for $n = 1$ LIV theories. Respective differences between ΛCDM and brane world models is almost 1 hour for $n = 2$ and 16 minutes for $n = 1$. These systematic differences get higher with redshift. The most pronounced is the difference in time delays between ΛCDM and Var Quintessence (i.e. the model with linear $w(z)$ functions with parameters best fitted to SNIa). The resulting mismatch between predicted time delays (from a source at $z = 3$) ranges from 1.25 hour in $n = 1$ theories to 6 hours in $n = 2$ LIV theories. Respective values for more distant source (at $z = 6$) are almost 4 hours ($n = 1$) and 27.5 hours.

Our results indicate that our ignorance concerning true model of dark energy in the universe is not able to spoil the utility of time delays in discriminating between $n = 1$ and $n = 2$ classes of LIV theories. However in each class of LIV theories it introduces an uncertainty at the level from 7% (ΛCDM, Quintessence, Chaplygin, braneworld) up to 35% (Var Quintessence) for sources at $z = 3$ (i.e. the most likely located GRBs). This translates into ranges 7% – 35% and 14% – 70% uncertainty for inferred bounds on $\xi_n E_{QG}$ in $n = 1$ and $n = 2$ cases respectively. For more distant sources this is respectively higher.

Therefore the conclusion is that better understanding of dark energy dominated Universe is crucial for testing LIV theories with cosmological sources like GRBs. Theoretically one may also invert this argument by saying that if LIV dispersion relation was proven experimentally and its parameters were constrained then time delays from GRBs could become a new kind of cosmological test.

Table 1: Expansion rates $H(z)$ in four models tested. The quantities Ω_i represent fractions of critical density currently contained in energy densities of respective components (like clumped pressure-less matter, Λ, quintessence, Chaplygin gas or brane effects).

Model	Cosmological expansion rate $H(z)$ (the Hubble function).
ΛCDM	$H^2(z) = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_\Lambda\right]$
Quintessence	$H^2(z) = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_Q (1+z)^{3(1+w)}\right]$
Var Quintessence	$H^2(z) = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_Q (1+z)^{3(1+w_0-w_1)} \exp(3w_1 z)\right]$
Chaplygin Gas	$H(z)^2 = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_{Ch} \left(A_0 + (1-A_0)(1+z)^{3(1+\alpha)}\right)^{\frac{1}{1+\alpha}}\right]$
Braneworld	$H(z)^2 = H_0^2 \left[\left(\sqrt{\Omega_m (1+z)^3 + \Omega_{r_c}} + \sqrt{\Omega_{r_c}}\right)^2\right]$

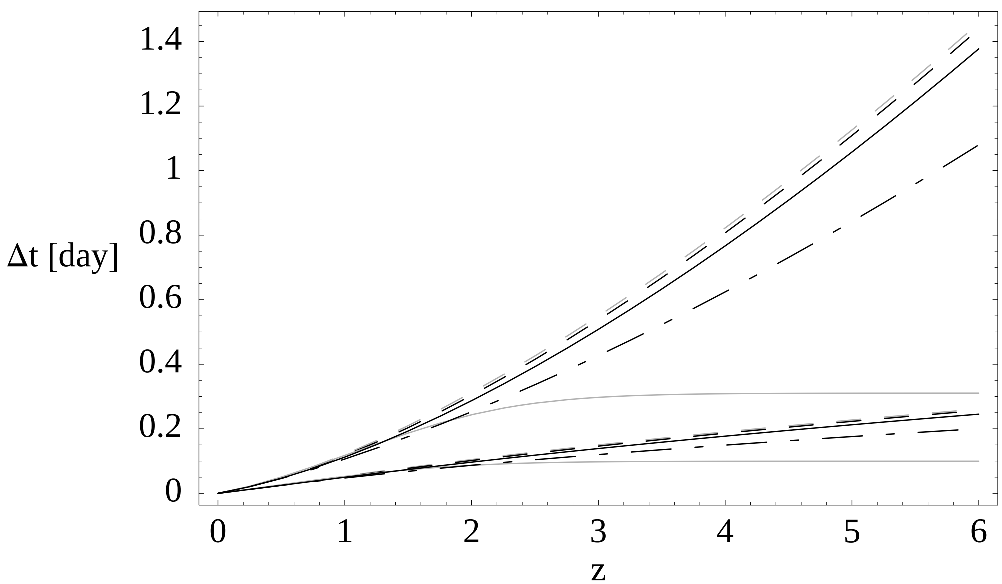

Figure 1: Observed time delays for 100 TeV neutrinos as a function of redshift in different dark energy scenarios (ΛCDM —light gray dashed line, quintessence — black dashed line, quintessence with varying E.O.S. — light gray solid line, brane world model — black solid line and Chaplygin gas scenario — dot-dashed line). Upper curves correspond to $n = 2$, $\xi_2 = 10^{-7}$, lower curves correspond to $n = 1$, $\xi_1 = 1$.

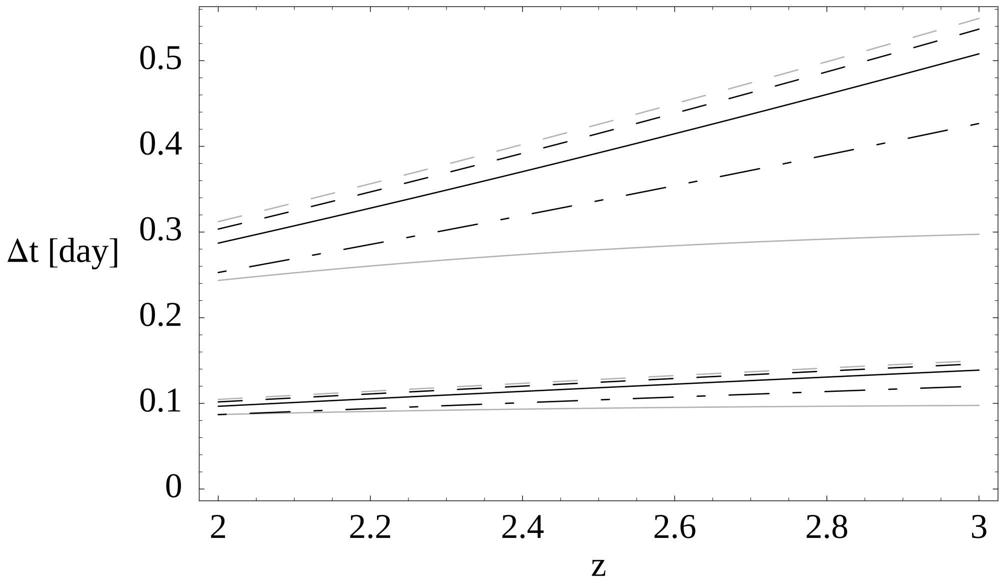

Figure 2: The same as Fig.1, in a restricted redshift range corresponding to the mode of GRB distribution.

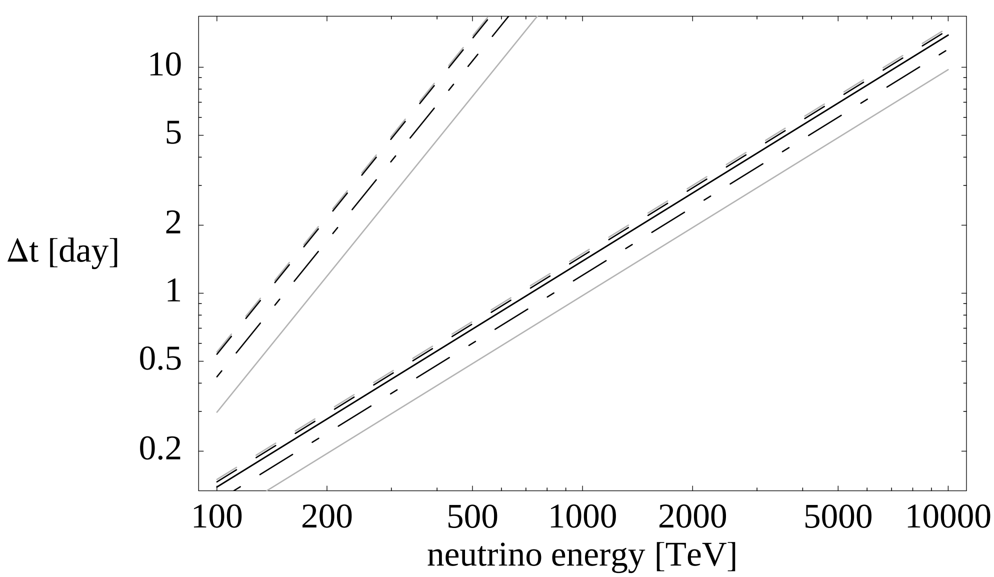

Figure 3: Time delays a a function of neutrino energy in different dark energy scenarios for a source located at $z = 3$. Left (steeper) family of curves corresponds to $n = 2$, $\xi_2 = 10^{-7}$ LIV theories, right family corresponds to $n = 1$, $\xi_1 = 1$ LIV theories.

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